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David Ruelle: The Mathematician's Brain

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A footnote in *The Mathematician' Brain* finds the distinguished mathematical physicist David Ruelle making this comment about Plato: "Of course, when you read Plato you should allow yourself to disagree with him.... But mostly one has the delightful impression of conversing with a very intelligent, open-minded, and pleasant man". It is a charming remark, and one that (*mutatis mutandis*) would equally apply to Ruelle himself. *The Mathematician's Brain* contains Ruelle's reflections on both the nature of mathematics and the lives of mathematicians, in roughly equal measure. Ruelle discusses mathematics as an activity rather than as a set of results, and opens a vista into the varied concerns of the practicing mathematician. For any reader interested as much in what being a mathematician is like as in what mathematics is, this book offers the inside scoop.

The most salient difference between Plato's style and Ruelle's, though, is this: Platonic dialogues, no matter how digressive they may seem at times, always remain focused on a single problem. Subsidiary topics are taken up in service of the main line of inquiry, whether that inquiry is ultimately conclusive or not. Ruelle, in contrast, has the spirit of the *flaneur*: he strolls through the mathematical landscape, pausing here and there, recounting anecdotes, taking up foundational issues for a while then putting them aside to wonder over the motivations, personalities, and lives of mathematicians. Although there are several recurrent topics, there is no overarching thesis being argued. The reader should approach the book in the same spirit, happy to turn from an exposition of algebraic geometry to a memoir of Ruelle's colleague Alexander Grothendieck without the expectation that the latter will ultimately shed light on the former. Sprinkled throughout the text one will encounter consideration of the status of set theory, category theory, and mathematical physics interspersed with *apercus* concerning the role of honors in mathematics, the nature of mathematical invention, and Freud's analysis of Leonardo's personality. Unlike Plato, Ruelle's digressions really do digress, and one should be along for the ride to enjoy the passing scenery more than being concerned about the final destination.

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But since a book review needs to focus on a central theme, even when the book reviewed does not, let me highlight a couple of recurring topics and make a few observations about them. Having so sympathetically described Plato the man, Ruelle warns the reader at several places that he is wary of the Platonistic account of mathematics: mathematical truths as possessing a reality independent of human thought and activity, mathematical truth as something *discovered* rather than *invented* or *constructed* by people. Since most mathematicians are depicted as Platonist at heart, this is an intriguing turn of events, and one wonders exactly how Ruelle will depart from the Platonist perspective. But the demurral, when it finally comes, is relatively mild: Ruelle does not seem to think that there is anything non-objective or non-absolute about mathematical *truth*, but rather that the sorts of problems human mathematicians find interesting and the conceptual apparatus they find most useful in constructing proofs are likely to be determined by the idiosyncrasies of the human mind. Alien mathematicians would not come to opposite conclusions from terrestrial ones, but they might spend their time on quite different topics and present their proofs using unfamiliar (and to us obscure) concepts. This constitutes a quite gentle correction to the Platonist view (if it is a correction at all), since the Platonist is more concerned about the status of mathematical truth than about the organization of mathematical activity.

Concerning the foundational issue itself, the situation is not so clear. Ruelle seems to accept that all provable mathematical claims can ultimately be derived from set theory—and more specifically from Zermelo-Fraenkel set theory together with the Axiom of Choice. Furthermore, he thinks that all such derivations could, in principle, be done by means of purely syntactically specifiable inference rules that could be checked mechanically by a computer. As far as I can tell (the book is a bit hazy on this point), Ruelle accepts the axioms of ZFC as all *true*, and the syntactically specified inferences as all *valid* (i.e. truth-preserving), so that all theorems so derived would express mathematical truths. He is aware, of course, that Gödel's theorem implies that theoremhood in this system cannot be equivalent to truth (there will be unprovable truths if the system is consistent), but seems to take this result in good Platonist fashion: the unprovable truths are, for all their unprovability, still *truths*.

Still, Ruelle is not quite as explicit on this point as one would like. He is aware that some mathematicians "frankly dislike the Axiom of Choice, and others make a mental note whether it is used in a given theory. But at this time most mathematicians consider that they obtain richer, more interesting mathematics with this axiom than without. This makes ZFC currently the standard basis for mathematics." This is all well and good as a *sociological* observation, but one wonders whether it is supposed to stand as a reply to the doubters of Choice. Such doubters (or at least some of them) might question whether the axiom itself is *true*, whether the results one derives from it are *reliable* and *accurate*, not whether the results are "richer" or "more interesting." One could presumably derive many rich and interesting claims from a false axiom, but that would not make for better mathematics. (One can evidently derive many interesting results from the Riemann Hypothesis, but that fact *alone* does not argue basing mathematical proofs on it, and calling it an *axiom* instead of a *hypothesis*.)

One could, of course, reject the discourse of "truth" and "falsehood" altogether, and portray mathematics as nothing but a game of symbol manipulation: one starts with certain strings of symbols (the "axioms") and certain specifiable rules for "deriving" new strings from old ones (the "inference rules"), and then sees what can be derived. Including or not including the "Axiom of Choice" would then just be a matter of defining the rules of two different games. But on this construal, the results of mathematical "proof" are just meaningless strings of symbols, and the issue of their truth and falsity does not arise.

It is entirely contrary to the spirit of this book to regard mathematics as such an empty game. Still, Ruelle returns repeatedly to an apparent tension between the completely rigorous, formalized derivation from ZFC on the one hand and the actual proofs of real mathematicians on the other. The latter use a rich supply of concepts and are presented in natural language, while the former can "be identified with the output of a Turing machine working forever at listing all of the consequences of the ZFC axioms". Ruelle seems to think that mechanical output is, in some sense, the ideal of mathematics, while the less formal mathematical arguments are forced on us by the peculiarities of the human mind, with its small memory and short attention span. But of course the Turing machine's activities, without humans to interpret the significance of the formal notation, do not produce meaningful claims at all: the Turing machine is not doing mathematics, it is just following a program. And the significance of the machine's output cannot be evaluated without a decision as to whether what the axioms *say* is true and whether the derivations preserve truth: the foundational issues are not removed by automating the activity.

It is true, as Ruelle says, that flesh-and-blood mathematicians hardly ever derive their results starting with ZFC, but it is not clear what the significance of this observation is supposed to be. If it is because they simply take for granted many results already known to be proven from ZFC (as Ruelle notes), then this is just a matter of being economical. If it is because their proof techniques *could not* be reformulated as derivations from ZFC, then it is a different matter altogether. Had Ruelle taken a bit more attention to separate these sorts of issues, the foundational picture he is presenting would have been clearer.

But all of this seems like nit-picking, foreign to the spirit of the book. It is only a very good book that stimulates discussion of foundational issues at all, and *The Mathematician's Brain* does that and much else beside. One finds a rich, multi-textured, human account of mathematics and mathematical life here, an account that makes one wish to spend an afternoon with the author, in pleasant conversation about whatever captures one's fancy at the moment.